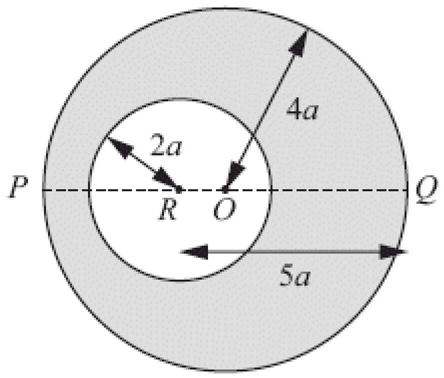
1.



A uniform circular disc has mass $4m$, centre O and radius $4a$. The line POQ is a diameter of the disc. A circular hole of radius $2a$ is made in the disc with the centre of the hole at the point R on PQ where $QR = 5a$, as shown in the diagram above.

The resulting lamina is free to rotate about a fixed smooth horizontal axis L which passes through Q and is perpendicular to the plane of the lamina.

(a) Show that the moment of inertia of the lamina about L is $69ma^2$.

(7)

The lamina is hanging at rest with P vertically below Q when it is given an angular velocity Ω. Given that the lamina turns through an angle $\frac{2\pi}{3}$ before it first comes to instantaneous rest,

(b) find Ω in terms of g and a.

(6)
(Total 13 marks)

2. A uniform lamina ABC of mass m is in the shape of an isosceles triangle with $AB = AC = 5a$ and $BC = 8a$.

(a) Show, using integration, that the moment of inertia of the lamina about an axis through A, parallel to BC, is $\frac{9}{2}ma^2$.

(6)

The foot of the perpendicular from A to BC is D. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through D and is perpendicular to the plane of the lamina. The lamina is released from rest when DA makes an angle α with the downward vertical. It is given that the moment of inertia of the lamina about an axis through A, perpendicular to BC and in the plane of the lamina, is $\frac{8}{3}ma^2$.

(b) Find the angular acceleration of the lamina when DA makes an angle θ with the downward vertical.

(8)

Given that α is small,

(c) fin d an approximate value for the period of oscillation of the lamina about the vertical.

(2)
(Total 16 marks)

3. A uniform circular disc has mass m, centre O and radius $2a$. It is free to rotate about a fixed smooth horizontal axis L which lies in the same plane as the disc and which is tangential to the disc at the point A. The disc is hanging at rest in equilibrium with O vertically below A when it is struck at O by a particle of mass m. Immediately before the impact the particle is moving perpendicular to the plane of the disc with speed $3\sqrt{(ag)}$. The particle adheres to the disc at O.

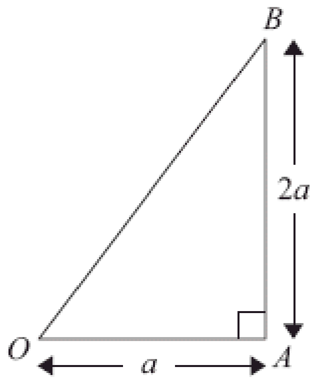
(a) Find the angular speed of the disc immediately after the impact.

(5)

(b) Find the magnitude of the force exerted on the disc by the axis immediately after the impact.

(6)
(Total 11 marks)

4.



A uniform lamina of mass M is in the shape of a right-angled triangle OAB. The angle OAB is 90°, $OA = a$ and $AB = 2a$, as shown in the diagram above.

(a) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina OAB is free to rotate about a fixed smooth horizontal axis along the edge OA and hangs at rest with B vertically below A. The lamina is then given a horizontal impulse of magnitude J. The impulse is applied to the lamina at the point B, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(b) find an expression for J, in terms of M, a and g.

(7)
(Total 13 marks)

5. A pendulum consists of a uniform rod AB, of length $4a$ and mass $2m$, whose end A is rigidly attached to the centre O of a uniform square lamina $PQRS$, of mass $4m$ and side a. The rod AB is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through B. The axis L is perpendicular to AB and parallel to the edge PQ of the square.

(a) Show that the moment of inertia of the pendulum about L is $75ma^2$.

(4)

The pendulum is released from rest when BA makes an angle α with the downward vertical through B, where $\tan \alpha = \frac{7}{24}$. When BA makes an angle θ with the downward vertical through B, the magnitude of the component, in the direction AB, of the force exerted by the axis L on the pendulum is X.

(b) Find an expression for X in terms of m, g and θ.

(9)

Using the approximation $\theta \approx \sin \theta$,

(c) find an estimate of the time for the pendulum to rotate through an angle α from its initial rest position.

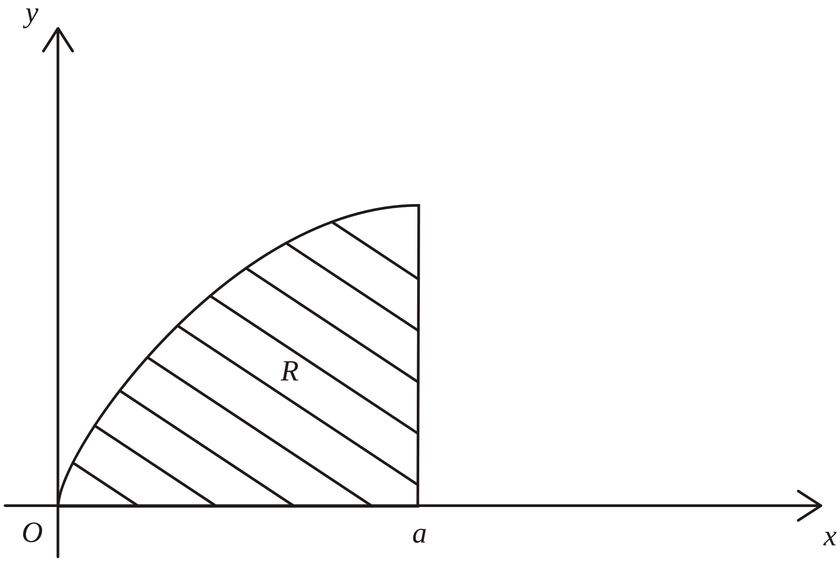
(6)
(Total 19 marks)

6. A uniform solid right circular cylinder has mass M, height h and radius a. Find, using integration, its moment of inertia about a diameter of one of its circular ends.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius a, about a diameter is* $\frac{1}{4}ma^2$.]

(Total 10 marks)

7.

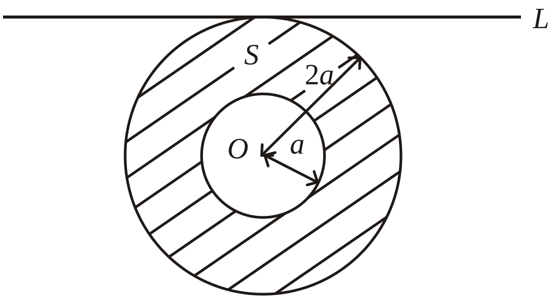


A region R is bounded by the curve $y^2 = 4ax$ ($y > 0$), the x-axis and the line $x = a$($a > 0$), as shown in the diagram above. A uniform solid S of mass M is formed by rotating R about the x-axis through 360°. Using integration, prove that the moment of inertia of S about the x-axis is $\frac{4}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.)

(Total 7 marks)

8.



A lamina S is formed from a uniform disc, centre O and radius $2a$, by removing the disc of centre O and radius a, as shown in the diagram above. The mass of S is M.

(a) Show that the moment of inertia of S about an axis through O and perpendicular to its plane is $\frac{5}{2}Ma^2$.

(3)

The lamina is free to rotate about a fixed smooth horizontal axis L. The axis L lies in the plane of S and is a tangent to its outer circumference, as shown in the diagram above.

(b) Show that the moment of inertia of S about L is $\frac{21}{4}Ma^2$.

(4)

S is displaced through a small angle from its position of stable equilibrium and, at time $t = 0$, it is released from rest. Using the equation of motion of S, with a suitable approximation,

(c) find the time when S first passes through its position of stable equilibrium.

(6)
(Total 13 marks)

9. (a) Prove, using integration, that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis perpendicular to the rod through one end is $\frac{4}{3}ma^2$.

(3)

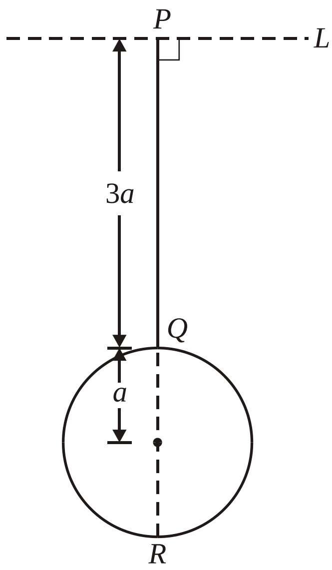
(b) Hence, or otherwise, find the moment of inertia of a uniform square lamina, of mass M and side $2a$, about an axis through one corner and perpendicular to the plane of the lamina.

(3)
(Total 6 marks)

10. A uniform circular disc has radius a and mass m. Prove, using integration, that the moment of inertia of the disc about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(Total 5 marks)

11.



A thin uniform rod PQ has mass m and length $3a$. A thin uniform circular disc, of mass m and radius a, is attached to the rod at Q in such a way that the rod and the diameter QR of the disc are in a straight line with $PR = 5a$. The rod together with the disc form a composite body, as shown in the diagram. The body is free to rotate about a fixed smooth horizontal axis L through P, perpendicular to PQ and in the plane of the disc.

(a) Show that the moment of inertia of the body about L is $\dfrac{77ma^2}{4}$.

(7)

When PR is vertical, the body has angular speed ω and the centre of the disc strikes a stationary particle of mass $\frac{1}{2}m$. Given that the particle adheres to the centre of the disc,

(b) find, in terms of ω, the angular speed of the body immediately after the impact.

(4)
(Total 11 marks)

12. A uniform lamina of mass m is in the shape of an equilateral triangle ABC of perpendicular height h. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis L through A and perpendicular to the lamina.

(a) Show, by integration, that the moment of inertia of the lamina about L is $\dfrac{5mh^2}{9}$.

(9)

The centre of mass of the lamina is G. The lamina is in equilibrium, with G below A, when it is given an angular speed $\sqrt{\left(\frac{6g}{5h}\right)}$.

(b) Find the angle between AG and the downward vertical, when the lamina first comes to rest.

(5)

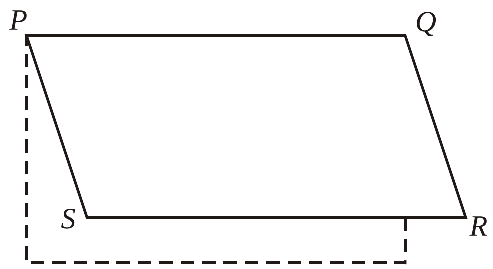
(c) Find the greatest magnitude of the angular acceleration during the motion.

(3)
(Total 17 marks)

13. A uniform lamina of mass m is in the shape of a rectangle $PQRS$, where $PQ = 8a$ and $QR = 6a$.

(a) Find the moment of inertia of the lamina about the edge PQ.

(2)



The flap on a letterbox is modelled as such a lamina. The flap is free to rotate about an axis along its horizontal edge PQ, as shown in the diagram above. The flap is released from rest in a horizontal position. It then swings down into a vertical position.

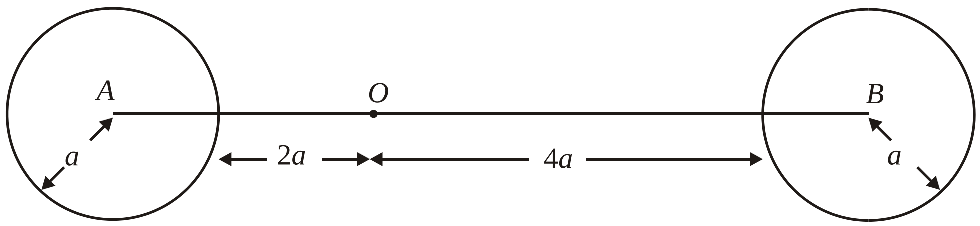
(b) Show that the angular speed of the flap as it reaches the vertical position is $\sqrt{\left(\frac{g}{2a}\right)}$.

(3)

(c) Find the magnitude of the vertical component of the resultant force of the axis PQ on the flap, as it reaches the vertical position.

(4)
(Total 7 marks)

14.



A body consists of two uniform circular discs, each of mass m and radius a, with a uniform rod. The centres of the discs are fixed to the ends A and B of the rod, which has mass $3m$ and length $8a$. The discs and the rod are coplanar, as shown in the diagram above. The body is free to rotate in a vertical plane about a smooth fixed horizontal axis. The axis is perpendicular to the plane of the discs and passes through the point O of the rod, where $AO = 3a$.

(a) Show that the moment of inertia of the body about the axis is $54ma^2$.

(6)

The body is held at rest with AB horizontal and is then released. When the body has turned through an angle of 30°, the rod AB strikes a small fixed smooth peg P where $OP = 3a$. Given that the body rebounds from the peg with its angular speed halved by the impact,

(b) show that the magnitude of the impulse exerted on the body by the peg at the impact is $9m\sqrt{\left(\frac{5ga}{6}\right)}$.

(10)
(Total 16 marks)

15. (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius a, about an axis through its centre O perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(4)

The line AB is a diameter of the disc and P is the mid-point of OA. The disc is free to rotate about a fixed smooth horizontal axis L. The axis lies in the plane of the disc, passes through P and is perpendicular to OA. A particle of mass m is attached to the disc at A and a particle of mass $2m$ is attached to the disc at B.

(b) Show that the moment of inertia of the loaded disc about L is $\frac{21}{4}ma^2$.

(6)

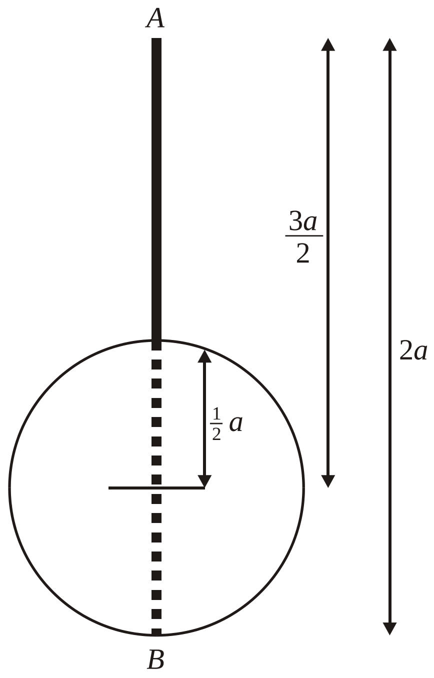
At time $t = 0$, PB makes a small angle with the downward vertical through P and the loaded disc is released from rest. By obtaining an equation of motion for the disc and using a suitable approximation,

(c) find the time when the loaded disc first comes to instantaneous rest.

(8)
(Total 18 marks)

16. (a) Show by integration that the moment of inertia of a uniform disc, of mass m and radius a, about an axis through the centre of disc and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(3)



A uniform rod AB has mass $3m$ and length $2a$. A uniform disc, of mass $4m$ and radius $\frac{1}{2}a$, is attached to the rod with the centre of the disc lying on the rod a distance $\frac{3}{2}a$ from A. The rod lies in the plane of the disc, as shown in the diagram above. The disc and rod together form a pendulum which is free to rotate about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the pendulum.

(b) Show that the moment of inertia of the pendulum about L is $\frac{27}{2}ma^2$.

(3)

The pendulum makes small oscillations about its position of stable equilibrium.

(c) Show that the motion of the pendulum is approximately simple harmonic, and find the period of the oscillations.

(6)
(Total 12 marks)

1. (a)

Mass of disc removed = m B1

$\frac{1}{2}4m(4a)^2 + 4m(4a)^2$ M1 A1

$\frac{1}{2}m(2a)^2 + m(5a)^2$ M1 A1

$I = \frac{1}{2}4m(4a)^2 + 4m(4a)^2 - (\frac{1}{2}m(2a)^2 + m(5a)^2)$ DM1

$= 69ma^2$ * A1 7

(b)

$4m.0 = 3m\bar{x} - ma$ M1

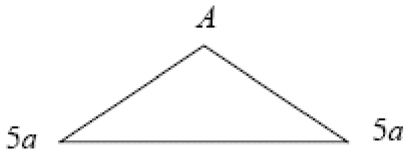
$\bar{x} = \frac{1}{3}a$ (from O) A1

$\frac{1}{2}69ma^2\Omega^2 = 3mg(4a - \frac{1}{3}a)(1 - \cos\frac{2\pi}{3})$ M1 A2

$\Omega = \sqrt{\frac{11g}{23a}}$ A1 6

[13]

2. (a)



$\delta A = \frac{8x}{3}\delta x$ M1 A1

$\delta m = \frac{8x}{3}\delta x.\frac{m}{12a^2}$ or $\delta m = \frac{8x}{3}\delta x.\rho$ DM1

$\delta I = \frac{8x}{3}\delta x.\frac{m}{12a^2}x^2 \quad (= \frac{2m}{9a^2}x^3\delta x)$ A1

$I = \int_0^{3a} \frac{2m}{9a^2}x^3 \, dx$ M1

$= \frac{2m}{9a^2}\left[\frac{x^4}{4}\right]_0^{3a}$

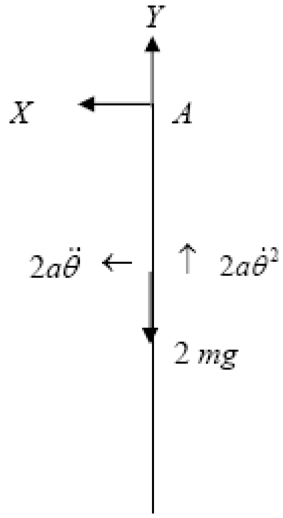
$= \frac{9ma^2}{2}$* A1 6

(b)	$I_A = \frac{9ma^2}{2} + \frac{8ma^2}{3} = \frac{43ma^2}{6}$ (perp axes rule)	M1 A1	
	$I_A = I_G + m(2a)^2$ (parallel axes rule)	DM1 A1	
	$I_D = I_G + ma^2$ (parallel axes rule)	A1	
	$I_D = \frac{43ma^2}{6} - 3ma^2 = \frac{25ma^2}{6}$	A1	
	$mga\sin\theta = -\frac{25ma^2}{6}\ddot{\theta}$	M1	
	$\ddot{\theta} = -\frac{6g}{25a}\sin\theta$	A1	8
(c)	For small θ, $\ddot{\theta} = -\frac{6g}{25a}\theta$ SHM	M1	
	$T = 2\pi\sqrt{\frac{25a}{6g}} = 5\pi\sqrt{\frac{2a}{3g}}$	A1	2

[16]

3.	(a)	MI of disc about L = $\frac{1}{4}m(2a)^2 + m(2a)^2 = 5ma^2$	M1 A1	
		CAM: $m3\sqrt{ag}.2a = (5ma^2 + m(2a)^2)\omega$	M1 A1 ft	
		$\omega = \frac{2}{3}\sqrt{\frac{g}{a}}$	A1	5

(b)



$M(A),\ 0 = I\ddot{\theta}$	B1	
$\ddot{\theta} = 0$		
$R(\leftarrow), X = 2m2a\ddot{\theta} = 0$	B1	
$R(\uparrow), Y - 2mg = 2m2a\dot{\theta}^2$	M1 A1	
$Y = 2mg + 4ma\dfrac{4g}{9a}$	DM1	
$= \dfrac{34mg}{9}$	A1	6

[11]

4. (a)

$\delta m = \dfrac{2Mx\delta x}{a^2}$	M1 A1	
$\delta I = \dfrac{1}{3}\dfrac{2Mx\delta x}{a^2}(2x)^2$	M1 A1	
$I = \displaystyle\int_0^a \dfrac{8Mx^3\,\mathrm{d}x}{3a^2}$	DM1	
$= \dfrac{8M}{3a^2}\left[\dfrac{x^4}{4}\right]_0^a$		
$= \dfrac{2}{3}Ma^2$ *	A1	6

	(b)	$J.2a = \frac{2}{3}Ma^2\omega$	M1 A1	
		$\frac{1}{2}\frac{2}{3}Ma^2\omega^2 = Mg\frac{2a}{3}(1+\cos 60°)$	M1 A2	
		solving for J	DM1	
		$J = M\sqrt{\frac{ag}{3}}$	A1	7

[13]

5.	(a)	$\frac{1}{3}2m(4a)^2 + \frac{1}{12}4ma^2 + 4m(4a)^2$	B1 M1 A1	
		$= \frac{32}{3}ma^2 + \frac{1}{3}ma^2 + 64ma^2$		
		$= 75ma^2$ *	A1	4
	(b)	$\frac{1}{2}\ 75ma^2\omega^2 = 2mg2a(\cos\theta - \cos\alpha) + 4mg4a(\cos\theta - \cos\alpha)$	M1 A2	
		$a\omega^2 = \frac{8}{15}g(\cos\theta - \frac{24}{25}) = \frac{8}{375}g(25\cos\theta - 24)$	A1	
		$X - 6mg\cos\theta = 2m2a\omega^2 + 4m4a\omega^2 = 20ma\omega^2$	M1 A2	
		$X = 6mg\cos\theta + 20m\frac{8}{375}g(25\cos\theta - 24)$	D M1	
		$= \frac{50mg\cos\theta}{3} - \frac{256mg}{25}$	A1	9

(c)	$-2mg\ 2a\sin\theta - 4mg\ 4a\sin\theta = 75ma^2\ddot{\theta}$	M1 A1	
	$\ddot{\theta} = -\dfrac{4g}{15a}\sin\theta$	A1	
	$\approx -\dfrac{4g}{15a}\theta$, SHM	M1	
	Time = $\dfrac{1}{4}2\pi\sqrt{\dfrac{15a}{4g}}$	M1	
	$\dfrac{\pi}{4}\sqrt{\dfrac{15a}{g}}$	A1	6

[19]

6.	$\delta m = \pi a^2\delta x.\dfrac{M}{\pi a^2 h} = \dfrac{M\delta x}{h}$	M1 A1	
	$\delta I = \dfrac{1}{4}\delta m.a^2 + \delta m.x^2$	M1 A1	
	$= \dfrac{M}{4h}(a^2 + 4x^2)\delta x$	M1 A1	
	$I = \int_0^h \dfrac{M}{4h}(a^2 + 4x^2)\mathrm{d}x$	M1 A1	
	$= \dfrac{M}{4h}\left[a^2x + \dfrac{4}{3}x^3\right]_0^h$	M1	
	$= \dfrac{M}{4}(a^2 + \dfrac{4}{3}h^2)$		
	$= \dfrac{M}{12}(3a^2 + 4h^2)$	A1	10

[10]

7.		$V = \pi\int_0^a 4ax\,\mathrm{d}x$	M1	
		$= 2\pi a^3$	A1	
		$\delta m = \dfrac{M}{2\pi a^3}\pi 4ax\delta x\left(= \dfrac{2M}{a^2}x\delta x\right)$	M1	
		$\delta \mathrm{I} = \dfrac{1}{2}\dfrac{2M}{a^2}x\delta x.y^2 = \dfrac{4M}{a}x^2\delta x$	M1A1	
		$I = \dfrac{4M}{a}\int_0^a x^2\,\mathrm{d}x = \dfrac{4}{3}Ma^2$	DM1A1	7

[7]

8.	(a)	$I_O = \dfrac{M}{3\pi a^2}\left(\dfrac{\pi}{2}(2a)^2(2a)^2 - \dfrac{\pi}{2}(a)^2(a)^2\right)$	M1A1	
		$= \dfrac{5Ma^2}{2}*$	A1	3
	(b)	$I_{\text{diameter}} = \dfrac{1}{2}\dfrac{5Ma^2}{2}$ (perp. axes)	M1A1	
		$I_L = \dfrac{5Ma^2}{4} + M(2a)^2$ (parallel axes)	M1	
		$= \dfrac{21Ma^2}{4}$	A1	4
	(c)	M(L), $-Mg2a\sin\theta = \dfrac{21Ma^2}{4}\ddot{\theta}$	M1A1	
		$\sin\theta \approx \theta \Rightarrow \ddot{\theta} = -\dfrac{8g}{21a}\theta$, so SHM	DM1A1	
		Time = $\dfrac{1}{4}2\pi\sqrt{\dfrac{21a}{8g}}$	DM1	
		$= \dfrac{\pi}{2}\sqrt{\dfrac{21a}{8g}}$	A1	6

[13]

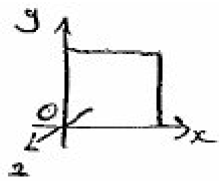
9. (a)

$I = \int_0^{2a} \frac{m}{2a} x^2 dx$ M1

$= \frac{m}{2a}\left[x^3/3 \right]_0^{2a}$ A1

$= \frac{4}{3} ma^2$ * A1 3

(b)

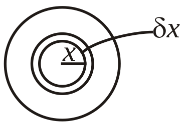


$I_x = I_y = \frac{4}{3} mc^2$ M1

$I_z = I_x + I_y = \frac{8}{3} mc^2$ ($\perp^{ar}$ axes) M1 A1 3

[6]

10.

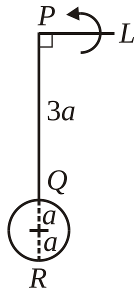


$\delta m = \frac{m}{\pi a^2} . 2\pi \times \delta x$ M1 A1

$\delta I = \frac{m}{\pi a^2} . 2\pi x^3 \delta x$ M1

$I = \frac{2m}{a^2} \int_0^a x^3 \mathrm{d}x = \underline{\frac{1}{2} ma^2}$ M1 A1 5

11. (a)



DISC: $I_{\text{diam}} = \frac{1}{2}\left(\frac{1}{2}ma^2\right) = \frac{1}{4}ma^2$ M1 A1

$I_L = \frac{1}{4}ma^2 + m(4a)^2$ M1 A1

$= \frac{65}{4}ma^2$

ROD: $I_L = \frac{4}{3}\left(\frac{3a}{2}\right)^2 = 3ma^2$ B1

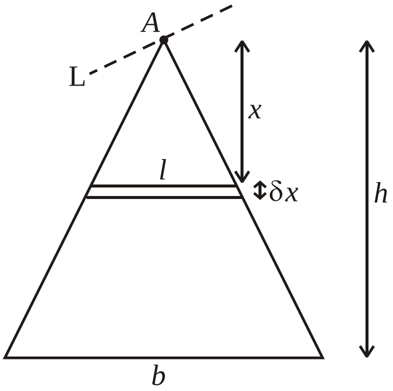
$I_{\text{TOTAL}} = \frac{65}{4}ma^2 + 3ma^2 = \underline{\frac{77}{4}ma^2}$ (*) M1 A1 7

(b) CAM: $\frac{77}{4}ma^2w = \left[\frac{77}{4}ma^2 + \frac{1}{2}m(4a)^2\right]w'$ M1 A1 A1

$\Rightarrow \underline{w' = \frac{77}{109}w}$ A1 4

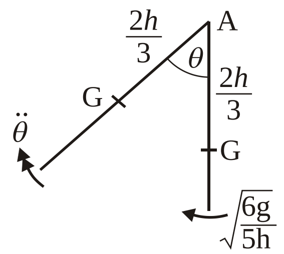
[11]

12. (a)



$\frac{l}{x} = \frac{b}{h}\left(= \frac{2}{\sqrt{3}}\right)$	M1
$\Rightarrow l = \frac{bx}{h} = \frac{2x}{\sqrt{3}}$	A1
$\rho = \frac{m}{\frac{1}{2}bh} = \frac{2m}{bh}$	
$\delta m = \frac{bx}{h}.\delta x.\frac{2m}{bh} = \frac{2m}{bh} = \frac{2m}{h^2}x\delta x$	M1
$\delta T_L \cong \frac{1}{3}\delta m\left(\frac{l}{2}\right)^2 + \delta m x^2$	M1 M1 A1
$= \frac{1}{12}(l^2 + 12x^2)\delta m$	
$= \frac{1}{12}\left(\frac{4x^2}{3} + 12x^2\right)\frac{2mx}{h^2}\delta x$	
$= \frac{80}{36}\frac{m}{h^2}x^3\delta x$	
$= \frac{20}{9}\frac{m}{h^2}x^3\delta x$	A1
$I_L = \frac{20}{9}\frac{m}{h^2}\int_0^h x^3\delta x$	M1
$= \frac{20}{9}\frac{m}{h^2}.\frac{h^4}{4}$	
$= \frac{5mh^2}{9}$ (*)	A1 9

(b)



Energy:

$\frac{1}{2}.\frac{5m}{9}h^2\dot{\theta}^2 = mg\frac{2h}{3}(1-\cos\theta)$

$\Rightarrow \frac{1}{2}.\frac{5mh^2}{9}\frac{6g}{5h} = mg\frac{2h}{3}(1-\cos\theta)$ M1 A1 A1

$\cos\theta = \frac{1}{2}$ M1

$\underline{\theta = \frac{\pi}{3}}$ (60°) A1 5

(c) $M(A)$: $-mg.\frac{2h}{3}\ \sin 60 = \frac{5mh^2}{9}\ddot{\theta}$ M1 A1ft on θ

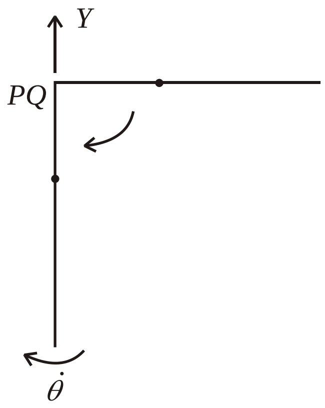
(max L^r acc^n when at rest)

$|\dot{\theta}|_{\max} = \frac{3\sqrt{3g}}{5h}$ A1 3

[17]

13. (a) $I_{PQ} = \frac{4}{3}m(3a)^2 = \underline{12\,ma^2}$ M1 A1 2

(b)



Energy: $\frac{1}{2} \times 12\,ma^2 \times \dot{\theta}^2 = mg \times 3a$ M1 A1 ft

$\Rightarrow \underline{\dot{\theta} = \sqrt{\left(\frac{g}{2a}\right)}}$ (*) A1 3

(c) $R(\uparrow)$: $Y - mg = m \times 3a\dot{\theta}^2$ M1 A1

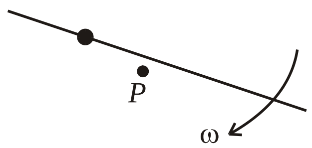
$Y = \text{mg} + \text{m} \times 3\text{a} \times \frac{g}{2a} = \underline{\frac{5}{2}mg}$ M1 A1 4

[9]

14. (a) $I = \frac{1}{2}ma^2 + m(3a)^2 + \frac{1}{2}ma^2 + m(5a)^2,$ M1 A1 A1

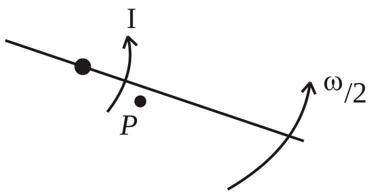
$(+)\frac{1}{3}(3m)(4a)^2 + 3ma^2$ M1 A1

$= \underline{54\,ma^2}$ (*) A1 6



$\frac{1}{2} \times 54ma^2 \times \omega^2 = mg \times \frac{5a}{2} + 3mg \times \frac{a}{2} - mg \times \frac{3a}{2}$ M1 A2, 1, 0

$\Rightarrow \omega = \sqrt{\frac{5g}{54a}} = \frac{1}{3}\sqrt{\frac{5g}{6a}}$ M1 A1



$3a \times \mathrm{I} = 54ma^2 \left[\frac{1}{2}\sqrt{\frac{5g}{54a}} - \left(-\sqrt{\frac{5g}{54a}}\right)\right]$ M1 A2, 1, 0 ft

$\mathrm{I} = \frac{54ma^2}{3a} \times \frac{3}{2}\sqrt{\frac{5g}{54a}}$

$= \frac{54ma^2}{3a} \times \frac{3}{2} \times \frac{1}{3}\sqrt{\frac{5g}{6a}}$

$= \underline{9m\sqrt{\frac{5ga}{6}}}$ M1 A1 10

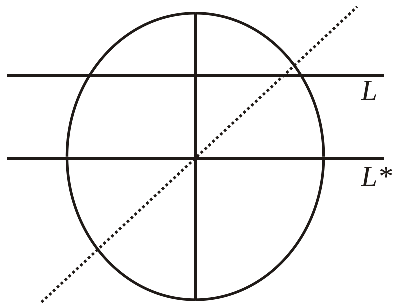
[16]

15. (a) $(\delta I) = (\rho)2\pi r\delta r.r^2$ M1

Using $(\rho) = \frac{m}{\pi a^2}$ M1

Completion: $\mathrm{I} = \frac{2m}{a^2}\left[\frac{r^4}{4}\right]_0^a = \frac{1}{2}ma^2$ (*) M1 A1 4

(b)



Disc: Use of $\perp^r$ axis theorem to find I_{L*} M1

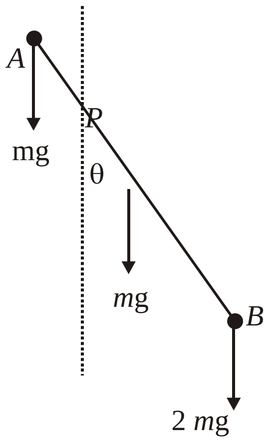
$I_{L*} = \frac{1}{2}\ (\frac{1}{2}\ ma^2) = \frac{1}{4}\ ma^2$ A1

Use of parallel axis theorem

$I_L = \frac{1}{4}\ ma^2 + m\left(\frac{a}{2}\right)^2 = \frac{1}{2}ma^2$ M1 A1

For loaded disc: $I = \frac{1}{2}ma^2 + m\left(\frac{a}{2}\right)^2 + 2m\left(\frac{3a}{2}\right)^2 = \frac{21}{4}ma^2$ (*) M1 A1 cso 6

(c)



$$I\ddot{\theta} = \left\{ mg\left(\frac{a}{2}\right)\sin\theta - mg\left(\frac{a}{2}\right)\sin\theta - 2mg\left(\frac{3a}{2}\right)\sin\theta \right\}$$ M1 A1 A1

[A1 for signs, A1 "terms"]

$$\left[\frac{21}{4}ma^2\ddot{\theta} = -3mga\sin\theta\right]$$

For small angles $\theta \approx \sin\theta \Rightarrow \frac{21}{4}ma^2\ddot{\theta} = -3mga\theta$ M1

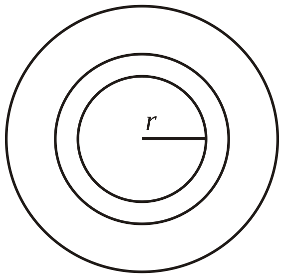
$\ddot{\theta} = -\frac{4g}{7a}\theta$ A1 ft

$\Rightarrow$ SHM with $\omega^2 = \frac{4g}{7a}$ M1

Time = $\frac{\pi}{\omega}$; $= \pi\sqrt{\frac{7a}{4g}}$ or $\frac{\pi}{2}\sqrt{\frac{7a}{g}}$ M1; A1 8

[18]

16. (a)



MI of element = $2\pi\rho r\delta r \times r^2$ M1

$m = \pi\rho a^2$

$\Rightarrow \text{I} = \dfrac{2m}{a^2}\displaystyle\int_0^a r^3\,\mathrm{d}r$ M1

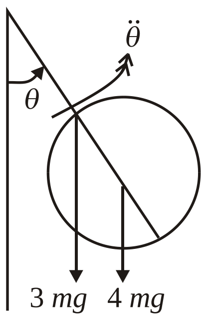
$= \dfrac{2m}{a^2}\left[\dfrac{r^4}{4}\right]_0^a = \tfrac{1}{2}ma^2$ A1 3

(b) $\text{I} = \text{I}_{\text{rod}} + \text{I}_{\text{disc}} = \tfrac{4}{3} \times 3m \times a^2, + \tfrac{1}{2} \times 4m\left(\dfrac{a}{2}\right)^2 + 4m\left(\dfrac{3a}{2}\right)^2$ B1, M1

$= 4ma^2 + \dfrac{ma^2}{2} + 9ma^2$

$= \tfrac{27}{2}ma^2$ A1 3

(c)



$\tfrac{27}{2}ma^2\,\ddot{\theta} = -3mg \times a\sin\theta - 4mg \times \dfrac{3a}{2}\sin\theta$ M1 A2, 1, 0

$= -9mga\sin\theta$

$\ddot{\theta} = -\dfrac{2g}{3a}\sin\theta$

Small oscillations $\Rightarrow \sin\theta \approx \theta$	M1	
$\Rightarrow \ddot{\theta} = -\frac{2g}{3a}\theta$ which is SHM	A1	
$T = 2\pi\sqrt{\frac{3a}{2g}}$	A1	6

[12]

1. In part (a) many correct solutions were seen. Errors arose in the use of the parallel axes rule. Despite distances being quite clearly given and marked on the diagram some candidates attempted to use $3a$, perhaps misreading the question. A few failed to use the parallel axes rule at all. It was surprising to see such basic errors in what was a fairly standard problem involving a change of axis for a moment of inertia.

In part (b) most candidates realised that an energy equation was required. A surprisingly large number, however, failed to realise that the position of the centre of mass for the lamina was required in the calculation of Potential Energy. Another common error was to use $4mg$ rather than $3mg$ for the weight of the lamina. It was surprising to see a number misquoting the expression for Kinetic Energy – sometimes omitting the $\frac{1}{2}$ or failing to square the Ω.

2. This question proved to be too difficult for many. Few completely correct solutions were seen.

Part (a) was very poorly answered. In order to find the mass of a strip, the ratio of base to height of the triangle was required. The height was easily found using a 3, 4,5 triangle yet a number of candidates made errors here with $\frac{3}{5}$ rather than $\frac{4}{3}$ seen quite often. Many candidates were unable to use appropriate methods for calculating the moment of inertia of a strip about the required axis through *A*. Many tried to use an axis through *BC* instead. A number used an incorrect density, failing to understand that *m* was the mass for the whole triangle and not half of it. Methods used were often very difficult to follow. There was often little indication of what the candidate was trying to do.

For part (b), very many thought that $\frac{8}{3}ma^2$ was the moment of inertia required in their solution. Very few realised that they needed to use this, together with the answer from part (a), and the perpendicular axes rule. Of those who did, few appreciated that they also needed to use the parallel axes rule, in order to find the moment of inertia about the axis required. Using their moment of inertia, in an equation of motion, to find the angular acceleration also proved to be a stumbling block for many, with $2a$ rather than a often seen. A significant number chose, instead, to differentiate an energy equation and both methods were generally used correctly by those who got this far.

In part (c) many candidates failed to write down the actual SHM equation for $\ddot{\theta}$ in terms of θ before writing down the periodic time, thereby losing all the marks.

3. A fairly high number of good attempts to this question were seen.

In part (a) most candidates used the correct moment of inertia for an axis tangential to the disc and some did then use the perpendicular axes rule followed by the parallel axes rule. Only a few, erroneously, used an axis perpendicular to the disc. Most of the errors in this part then resulted from trying to use energy conservation for an inelastic impact where energy was *not* conserved. Conservation of angular momentum was required here and those who used this were generally successful in scoring most of the available marks.

Very few completely correct solutions were seen to part (b). Some attempted to use energy conservation again. Of those who correctly realised that equations of motion were required, there were very many incomplete solutions with only a very small number correctly justifying the fact that the horizontal component of the force required was zero. Of those who correctly followed through a vertical equation of motion, quite a number failed to include the mass of the particle to make the total mass $2m$.

4. The simplest way to split the lamina into strips for part (a) was to use the hint in the question and have the strips parallel to AB. Those who used strips parallel to OA made the question more complicated and more difficult but were, nevertheless, often successful.

The commonest errors in the second part were either to get the position of the centre of mass wrong or to forget to multiply J by $2a$ when applying the rotational impulse-momentum principle.

5. The weakness of candidates when answering questions about rotational mechanics was shown by the large number who only attempted part (a) of this question. This sometimes involved blatant "fudging".

In part (b), the attempt at the energy equation was often reasonable, but using Newton's second law along the rod often contained dimensional errors, such as multiplying the component of the weight by a distance.

In the final part it was necessary to obtain an expression for the angular acceleration either by taking moments or by differentiating the energy equation from part (b). Without this starting point, it was not possible to use the approximation $\theta \approx \sin \theta$ and so few marks could be obtained. There are still a number of candidates who learn a formula for the period of a compound pendulum and this should be strongly discouraged as the questions are usually designed to test understanding and the ability to work from 1st principles.

6. Most candidates tried to use an increment of volume and then integrate, but a substantial minority did not use the parallel axes theorem to change to an axis through one of the ends of the cylinder, thus only gaining a possible 3 marks out of 10. A surprising number of candidates appeared to be considering a cone or a sphere. Some got their constants and variables mixed up, both in creating an expression for the moment of inertia of the increment and in integration.

7. A number of candidates correctly attempted to split the solid into discs with their centres on the x-axis and many arrived at the final answer. Some, however, assumed that the solid was a hemisphere when finding the density and an expression for y^2. The other common error was to take the moment of inertia of a disc to be $\frac{1}{2}\delta m x^2$ instead of $\frac{1}{2}\delta m y^2$.

8. The simplest (incorrect) method to get the printed answer in part (a) is by adding various pieces, each of mass M, which is what a significant number of the candidates did. The simplest correct method is by considering the ratio of the masses of the large disc and the small disc and hence the mass of the ring.

Part (b) using the perpendicular and parallel axis theorems was almost universally successful. In the third part, a few candidates seemed unaware of the use of the term "equation of motion" used in its rotational sense. Most, however, took moments about the axis and proceeded correctly. Common errors were to omit the minus sign, which was penalised when the period of SHM was needed, the 2 from $2a$, and thinking that half of the period was needed rather than a quarter of it. A few candidates ignored the instruction to use an equation of motion and started from a formula for the period of a compound pendulum, thus gaining no marks in part (c).

9. The first part proved to be an easy starter and was generally well answered. Part (b) was more demanding but nevertheless was well done by many, either by using the perpendicular axes rule directly or by starting from the centre of the square and using both the perpendicular axes and the parallel axes rules. A few candidates tried using integration but usually without success.

10. Nearly all candidates provided good proofs of this standard result and set out their working clearly. Only the weakest candidates failed to make good progress in this question.

11. This proved to be the most straightforward question on the paper with many fully correct solutions. The majority found the moment of inertia of the body correctly – the error for the minority was to use the moment of inertia of the disc about an axis through the centre rather than about the diameter. In part (b), a few candidates thought that energy was conserved, rather than angular momentum.

12. Part (a) proved to be beyond the majority of candidates. The proof was not well known and most candidates could not find the moment of inertia of an elementary strip. Use of the parallel and perpendicular axis theorems was needed but they were rarely seen. However, marks were gained in (b) and (c) since the inertia was given although candidates often lost marks because they used *mgh* instead of *2mgh/3* in the potential energy term and in the angular equation of motion.

13. This proved to be a very straightforward question for the vast majority of candidates and full marks here were regularly obtained. It was very pleasing to note the clear understanding shown of the general principles involved.

14. Most could verify that the moment of inertia of the given body was as stated. Most could also successfully find the angular speed of it just before it hit the peg. The last part of the question proved to be more challenging with relatively few fully correct solutions seen. Many confused linear and angular speeds and/or impulses, failing to take the moment of the impulse. Others too failed to realise that the angular velocity would change direction at the impulse and so had a wrong sign in their equation. The question as a whole proved to be a good discriminator, highlighting those with a clear understanding of the principles involved in angular motion.

15. Part (a) often provided candidates with 4 marks, although some candidates did need to "fudge" a little, and the approach of some candidates did involve a considerable amount of work. In part (b) good candidates did set out their working very clearly but often it was difficult to know what candidates were doing. A very frequent "solution", with no explanation, was

$I = \frac{1}{2}ma^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + 2m\left(\frac{3a}{2}\right)^2$ followed by $= \frac{21}{4}ma^2$, the given answer. For these candidates, who had not considered the need to apply the perpendicular axis theorem, the answer should have been $\frac{22}{4}ma^2$!

In part (c) it was quite common for candidates to omit one of the particles or make a sign error in the equation of motion. If the resulting equation, after the approximation of θ for $\sin\theta$, did not represent SHM the final three marks were not available. As stated in the introduction, if $T = 2\pi\sqrt{\frac{I}{mgh}}$ was used, without proof, then only 3 marks were available for a correct answer.

16. No Report available for this question.